

MEAN SQUARED ERROR ESTIMATORS OF SMALL AREA MEANS USING SURVEY WEIGHTS

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ABSTRACT

Using survey weights, You and Rao (2002) proposed a pseudo empirical best linear unbiased prediction (pseudo-EBLUP) estimator to estimate small area means under a nested error linear regression model. This estimator borrows strength across areas through the linking model, and makes use of survey weights to preserve benchmarking property in the sense that the estimators add up to a reliable direct estimator of a large area covering the small areas. In this paper, we derive a second order approximations to the mean squared error (MSE) of the pseudo-EBLUP estimator of a small area mean. Using this approximation, an estimator of MSE that is approximately unbiased is derived. Empirical results on the performance of the proposed approximation are also presented.

KEY WORDS: Benchmarking; Mean squared error estimation; Nested error regression model; Small area estimation; Survey weights.

RÉSUMÉ

You et Rao (2002) ont proposé un estimateur pseudo-EBLUP de la moyenne d'un petit domaine sous un modèle de régression linéaire à erreurs emboîtées en utilisant les poids d'échantillonnage. Cet estimateur gagne en précision sur l'ensemble des domaines au travers du modèle de lien, et utilise les poids de sondage pour préserver la propriété d'égalonage qui veut que les estimés somment à la valeur d'un estimateur direct d'une région plus large couvrant les petites régions. On a obtenu des approximations du deuxième ordre pour l'EQM du pseudo-EBLUP de la moyenne d'un petit domaine. En utilisant cette approximation, on obtient un estimateur de l'EQM qui est approximativement sans biais. Des résultats empiriques sur la performance des approximations proposées sont également présentés.

MOTS CLÉS: Benchmarking; estimation de l'erreur quadratique moyenne; estimation pour petits domaines; modèle de régression linéaire à erreurs emboîtées; poids d'échantillonnage.

1. INTRODUCTION

Small area estimation has received considerable attention due to growing demand for reliable small area statistics. Rao (2003) gives a comprehensive account of model-based small area estimation. In particular, nested error linear regression models are often used in small area estimation to obtain efficient model-based estimators of small area means. A nested error linear regression model was used by Battese, Harter and Fuller (1988) and Prasad and Rao (1990) for small area estimation, but they did not make use of the unit level survey weights. As a result, the estimators are not design consistent as the area sample size becomes large, unless the survey weights are equal within areas. On the other hand, direct estimators are design consistent, but fail to borrow strength from related small areas. You and Rao (2002) proposed a pseudo-EBLUP estimator for a small area mean that makes uses of survey weights. In addition, the method of You and Rao has a nice benchmarking property in the sense that the estimators automatically add up to a reliable direct estimator of a large area covering the small areas. However, they ignored a cross-product term in the MSE. As a result, this MSE estimator is not approximately unbiased. In this paper, we take account of this cross-product term and obtain an estimator of MSE of the pseudo-EBLUP that is approximately unbiased. Some empirical results are also presented.

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2. PSEUDO EBLUP

The nested error linear regression model of Battese et al. (1988) is given by

$$y_{ij} = x'_{ij}\beta + v_i + e_{ij}; j = 1, \dots, n_i; i = 1, \dots, m \quad (1)$$

where β is a $p \times 1$ vector, $v_i \stackrel{i.i.d.}{\sim} N(0, \sigma_v^2)$, $e_{ij} \stackrel{i.i.d.}{\sim} N(0, \sigma_e^2)$, n_i is the sample size within i -th area and m is the number of areas. The i -th area mean μ_i is given by $\mu_i = \bar{X}'_i\beta + v_i$, where \bar{X}_i is the known population mean of x_{ij} 's. You and Rao (2002) proposed a pseudo-EBLUP estimator of μ_i for model (1) that depends on the survey weights \tilde{w}_{ij} and satisfies the design consistency property. First assuming that the parameters β, σ_v^2 and σ_e^2 are known, a pseudo-EBLUP estimator of μ_i is obtained as

$$\tilde{\mu}_{iw}^B = \bar{X}'_i\beta + \gamma_{iw}(\bar{y}_{iw} - \bar{x}'_{iw}\beta) = \bar{X}'_i\beta + \tilde{v}_{iw}(\beta, \delta), \quad (2)$$

where $\bar{y}_{iw} = \sum_{j=1}^{n_i} w_{ij}y_{ij}$, $\bar{x}_{iw} = \sum_{j=1}^{n_i} w_{ij}x_{ij}$, $\gamma_{iw} = \sigma_v^2 / (\sigma_v^2 + \sigma_e^2 \alpha_i^2)$ with $\alpha_i^2 = \sum_{j=1}^{n_i} w_{ij}^2$,

$w_{ij} = \tilde{w}_{ij} / \sum_j \tilde{w}_{ij}$ and $\sum_j w_{ij} = 1$. The regression parameter β is then estimated for fixed $\delta = (\sigma_v^2, \sigma_e^2)'$ by solving the following survey-weighted estimating equation for β :

$$\sum_{i=1}^m \sum_{j=1}^{n_i} \tilde{w}_{ij} x_{ij} (y_{ij} - x'_{ij}\beta - \tilde{v}_{iw}(\beta, \delta)) = 0. \quad (3)$$

The solution to (3) is given by

$$\tilde{\beta}_w(\delta) = \left\{ \sum_{i=1}^m \sum_{j=1}^{n_i} \tilde{w}_{ij} x_{ij} (x_{ij} - \gamma_{iw} \bar{x}_{iw})' \right\}^{-1} \left\{ \sum_{i=1}^m \sum_{j=1}^{n_i} \tilde{w}_{ij} (x_{ij} - \gamma_{iw} \bar{x}_{iw}) y_{ij} \right\}. \quad (4)$$

To estimate σ_v^2 and σ_e^2 , You and Rao (2002) used the well known method of fitting-of- constants that can be simplified through a transformation method of Battese et al (1988). In this paper, we estimate δ by the maximum likelihood (ML) method to get the ML estimators $\hat{\delta} = (\hat{\sigma}_v^2, \hat{\sigma}_e^2)'$. The resulting estimator of β is given by $\hat{\beta}_w = \tilde{\beta}_w(\hat{\delta})$.

Substituting $\hat{\beta}_w, \hat{\sigma}_v^2$ and $\hat{\sigma}_e^2$ for β, σ_v^2 and σ_e^2 in (2), a pseudo-EBLUP (EB) estimator of μ_i is given by

$$\hat{\mu}_{iw} = \bar{X}'_i\hat{\beta}_w + \hat{\gamma}_{iw}(\bar{y}_{iw} - \bar{x}'_{iw}\hat{\beta}_w). \quad (5)$$

The pseudo-EBLUP estimator (5) is design consistent as n_i becomes large. Also, the pseudo-EBLUP estimators automatically satisfy the benchmarking property when the estimators $\hat{\mu}_{iw}$ are aggregated over i , assuming that the weights \tilde{w}_{ij} are calibrated to agree with the known population total N_i , i.e., $\sum_j \tilde{w}_{ij} = \tilde{w}_i = N_i$. We have $\sum_i N_i \hat{\mu}_{iw} = \hat{Y}_w + (X - \hat{X}_w)' \hat{\beta}_w$, where $\hat{Y}_w = \sum_i \sum_j \tilde{w}_{ij} y_{ij}$ and $\hat{X}_w = \sum_i \sum_j \tilde{w}_{ij} x_{ij}$ are the direct estimators of the large area totals Y and X respectively, and $\hat{Y}_w + (X - \hat{X}_w)' \hat{\beta}_w$ is the direct survey regression estimator of Y . Hence, the pseudo-EBLUP estimators $\hat{\mu}_{iw}$ satisfy the benchmarking property without any adjustment.

3. MSE ESTIMATION OF PSEUDO EBLUP

3.1 You and Rao Method

Ignoring the cross-product term $C_{iw}(\Theta) = E(\tilde{\mu}_{iw}^B - \mu_i)(\hat{\mu}_{iw} - \tilde{\mu}_{iw}^B)$ in $MSE(\hat{\mu}_{iw})$ where $\Theta = (\beta', \delta')'$, You and Rao (2002) obtained the following MSE approximation along the lines of Prasad and Rao (1990) :

$$MSE(\hat{\mu}_{iw}) \approx g_{1iw}(\delta) + g_{2iw}(\delta) + g_{3iw}(\delta), \quad (6)$$

where $g_{1iw}(\delta) = (1 - \gamma_{iw})\sigma_v^2$, $g_{2iw}(\delta) = (\bar{X}_i - \gamma_{iw}\bar{x}_{iw})'\Phi_w(\bar{X}_i - \gamma_{iw}\bar{x}_{iw})$ and $g_{3iw}(\delta) = \gamma_{iw}(1 - \gamma_{iw})^2\sigma_e^{-4}\sigma_v^{-2}h(\delta)$. Further,

$$\begin{aligned} \Phi_w = & \sigma_e^2 \left(\sum_{i=1}^m \sum_{j=1}^{n_i} x_{ij} t'_{ij} \right)^{-1} \left(\sum_{i=1}^m \sum_{j=1}^{n_i} t_{ij} t'_{ij} \right) \left(\sum_{i=1}^m \sum_{j=1}^{n_i} x_{ij} t'_{ij} \right)^{-1} \\ & + \sigma_v^2 \left(\sum_{i=1}^m \sum_{j=1}^{n_i} x_{ij} t'_{ij} \right)^{-1} \left[\sum_{i=1}^m \left(\sum_{j=1}^{n_i} t_{ij} \right) \left(\sum_{j=1}^{n_i} t_{ij} \right)' \right] \left(\sum_{i=1}^m \sum_{j=1}^{n_i} x_{ij} t'_{ij} \right)^{-1}, \end{aligned}$$

with $t_{ij} = \tilde{w}_{ij}(x_{ij} - \gamma_{iw}\bar{x}_{iw})$ and $h(\delta)$ is given by $h(\delta) = \sigma_e^4 \text{var}(\hat{\sigma}_v^2) + \sigma_v^4 \text{var}(\hat{\sigma}_e^2) - 2\sigma_v^2\sigma_e^2 \text{cov}(\hat{\sigma}_v^2, \hat{\sigma}_e^2)$. You and Rao (2002) also obtained an estimator of MSE, based on the approximation (6), as

$$mse_{YR1}(\hat{\mu}_{iw}) = g_{1iw}(\hat{\delta}) + g_{2iw}(\hat{\delta}) + 2g_{3iw}(\hat{\delta}). \quad (7)$$

If $C_{iw}(\Theta) = 0$, the MSE estimator (7) is approximately unbiased in the sense that its bias is of order $o(m^{-1})$, for large m , which is the case where $n_i \tilde{w}_{ij} = \tilde{w}_i$ or $w_{ij} = 1/n_i$.

An area-specific version of MSE estimator is given by

$$mse_{YR2}(\hat{\mu}_{iw}) \approx g_{1iw}(\hat{\delta}) + g_{2iw}(\hat{\delta}) + 2g_{3iw}^*(\hat{\delta}, y_i), \quad (8)$$

where $g_{3iw}^*(\delta, y_i) = \gamma_{iw}^2(1 - \gamma_{iw})^2\sigma_e^{-4}\sigma_v^{-4}h(\delta)(\bar{y}_{iw} - \bar{x}'_{iw}\tilde{\beta}_w)^2$.

3.2 The Proposed Method

We now derive an approximation to $MSE(\hat{\mu}_{iw})$ taking account of the cross-product term $C_{iw}(\Theta)$. We have

$$MSE(\hat{\mu}_{iw}) = g_{1iw}(\delta) + g_{2iw}(\delta) + g_{3iw}(\delta) + 2C_{iw}(\Theta) + o(m^{-1}). \quad (9)$$

To obtain an approximation for the cross-product term in (9), we make a further assumption on model parameters Θ .

We assume that the estimator $\hat{\Theta}$ of Θ is a solution to the estimating equations of the form

$$M(\Theta) = m^{-1} \sum_{i=1}^m a_{iw}(y_{i1}, \dots, y_{in_i}, \Theta) = 0, \quad (10)$$

where $a_{iw}(\cdot)$ is a $(p+2)$ -vector $E[a_{iw}(y_{i1}, \dots, y_{in_i}, \Theta)] = 0$ provided that Θ is the true vector of parameters.

Following Jiang and Lahiri (2005), we approximate the cross-product term, based on a Taylor expansion, as

$$\begin{aligned} C_{iw}(\Theta) \approx & E \left\{ \left[-2m^{-1} \left(\frac{\partial \tilde{\mu}_{iw}^B}{\partial \Theta'} \right) A^{-1} a_{iw}(\cdot) + \left(\frac{\partial \tilde{\mu}_{iw}^B}{\partial \Theta'} \right) A^{-1} \left(\frac{\partial M}{\partial \Theta'} \right) A^{-1} M(\Theta) - \frac{1}{2} \left(\frac{\partial \tilde{\mu}_{iw}^B}{\partial \Theta'} \right) A^{-1} \right. \right. \\ & \left. \left. \left(M'(\hat{\Theta}) A^{-T} E \left(\frac{\partial^2 M_j}{\partial \Theta \partial \Theta'} \right) A^{-1} M(\Theta) \right) + \frac{1}{2} M'(\Theta) A^{-T} \left(\frac{\partial^2 \tilde{\mu}_{iw}^B}{\partial \Theta \partial \Theta'} \right) A^{-1} M(\Theta) \right] (\tilde{v}_{iw} - v_i) \right\} \\ = & \tilde{C}_{iw}(\Theta), \end{aligned} \quad (11)$$

where $M(\hat{\Theta}) = m^{-1} \sum_{i=1}^m a_{iw}(y_{i1}, \dots, y_{in_i}, \hat{\Theta}) = 0$, $A = E(\partial M(\Theta) / \partial \Theta)'$, $A^{-T} = (A^{-1})'$,

$\tilde{v}_{iw} = \gamma_{iw}(\bar{y}_{iw} - \bar{x}'_{iw}\beta)$, M_j represents the j-th component of M and (b_j) represents a column vector with components b_j .

In the case of the nested error linear regression model (1), we have the survey weighted estimation equation (3) for β . The coefficient of a_{1iw} corresponding to β can be found from (3), as

$$a_{1iw} = \sum_{j=1}^{n_i} \tilde{w}_{ij} x_{ij} (y_{ij} - x'_{ij}\beta - \tilde{v}_{iw}(\beta, \delta)). \quad (12)$$

Maximum likelihood estimator of δ satisfies the estimating equations (10) based on the score functions. Score functions are given by

$$S_1(\beta, \delta) = \frac{-1}{2} \sum_{i=1}^m (\sigma_v^2 + \sigma_e^2 \alpha_i^2)^{-1} + \frac{1}{2} \sum_{i=1}^m (\bar{y}_{iw} - \bar{x}'_{iw}\beta)^2 (\sigma_v^2 + \sigma_e^2 \alpha_i^2)^{-2},$$

$$S_2(\beta, \delta) = \frac{-1}{2} \sum_{i=1}^m \alpha_i^2 (\sigma_v^2 + \sigma_e^2 \alpha_i^2)^{-1} + \frac{1}{2} \sum_{i=1}^m (\bar{y}_{iw} - \bar{x}'_{iw}\beta)^2 \alpha_i^2 (\sigma_v^2 + \sigma_e^2 \alpha_i^2)^{-2}.$$

Therefore, we may express the components of a_{iw} corresponding to σ_v^2 and σ_e^2 respectively as

$$a_{2iw} = -(\sigma_v^2 + \sigma_e^2 \alpha_i^2)^{-2} [(\bar{y}_{iw} - \bar{x}'_{iw}\beta)^2 - (\sigma_v^2 + \sigma_e^2 \alpha_i^2)], \quad (13)$$

$$a_{3iw} = -\alpha_i^2 (\sigma_v^2 + \sigma_e^2 \alpha_i^2)^{-2} [(\bar{y}_{iw} - \bar{x}'_{iw}\beta)^2 - (\sigma_v^2 + \sigma_e^2 \alpha_i^2)]. \quad (14)$$

Hence, (11) is well-defined using (12), (13) and (14) for a_{1iw} , a_{2iw} and a_{3iw} and (2) for $\tilde{\mu}_{iw}^B$.

Theorem 1: A second order approximation of the mean squared error (MSE) of the pseudo- EBLUP estimator $\hat{\mu}_{iw}$ is given by

$$MSE(\hat{\mu}_{iw}) = g_{1iw}(\delta) + g_{2iw}(\delta) + g_{3iw}(\delta) + 2\tilde{C}_{iw}(\Theta) + o(m^{-1}),$$

where $\tilde{C}_{iw}(\Theta)$ is given by (11).

It can be shown that $\tilde{C}_{iw}(\Theta) = 0$ when $n_i \tilde{w}_{ij} = \tilde{w}_i$ or $w_{ij} = 1/n_i$. We propose two approximately unbiased MSE estimators for the pseudo-EBLUP estimators. The first estimator of $MSE(\hat{\mu}_{iw})$ is

$$mse_{TR1}(\hat{\mu}_{iw}) = g_{1iw}(\hat{\delta}) - \left(\frac{\partial g_{1iw}}{\partial \Theta'} \right) \Big|_{\Theta=\hat{\Theta}} \hat{b}(\hat{\Theta}) + g_{2iw}(\hat{\delta}) + 2g_{3iw}(\hat{\delta}) + 2\tilde{C}_{iw}(\hat{\Theta}), \quad (15)$$

where $\hat{b}(\hat{\Theta}) = m^{-2} \sum_{i=1}^m \hat{A}^{-1} \left(\frac{\partial a_{iw}(\cdot)}{\partial \Theta'} \Big|_{\Theta=\hat{\Theta}} \right) \hat{A}^{-1} \hat{a}_{iw}(\cdot) - \frac{1}{2m^3} \hat{A}^{-1} \left(\sum_{i=1}^m \hat{a}'_{iw}(\cdot) \hat{A}^{-T} \left(\sum_{i=1}^m \frac{\partial^2 a_{jw}(\cdot)}{\partial \Theta \partial \Theta'} \Big|_{\Theta=\hat{\Theta}} \right) \hat{A}^{-1} \hat{a}_{iw}(\cdot) \right)$,

with $b(\hat{\Theta}) = E(\hat{\Theta} - \Theta)$, $a_{jw}(\cdot)$ represents the j-th component of $a_{iw}(\cdot)$ and as before, (b_j) represents a column vector with components b_j . The estimator $mse_{TR1}(\hat{\mu}_{iw})$ is approximately unbiased in the sense of Theorem 2.

Theorem 2:

$$E[mse_{TR1}(\hat{\mu}_{iw})] = MSE(\hat{\mu}_{iw}) + o(m^{-1}).$$

Following Jiang and Lahiri (2005), we propose a second estimator, an area-specific MSE estimator, as

$$mse_{TR2}(\hat{\mu}_{iw}) = g_{1iw}(\hat{\delta}) + g_{iw}(\hat{\Theta}) + 2\tilde{C}_{iw}(\hat{\Theta}) - \hat{B}_{iw}(\hat{\Theta}), \quad (16)$$

where $g_{iw}(\hat{\Theta}) = tr\{\hat{V}(\hat{\Theta})E[(\frac{\partial \tilde{\mu}_{iw}^B}{\partial \Theta})(\frac{\partial \tilde{\mu}_{iw}^B}{\partial \Theta})']|_{\Theta=\hat{\Theta}}\}$, $\hat{B}_{iw}(\hat{\Theta}) = (\frac{\partial g_{1iw}}{\partial \Theta'})|_{\Theta=\hat{\Theta}} \hat{b}(\hat{\Theta})$

$$+ \frac{1}{2}tr[\hat{V}(\hat{\Theta})(\frac{\partial^2 g_{1iw}}{\partial \Theta \partial \Theta'})|_{\Theta=\hat{\Theta}}] \quad \text{with} \quad \hat{V}(\hat{\Theta}) = m^{-2} \hat{A}^{-1}(\hat{\Theta}) \{ \sum_{i=1}^m \hat{a}_{iw}(\cdot) \hat{a}'_{iw}(\cdot) \} (\hat{A}^{-1}(\hat{\Theta}))'$$

$$\hat{A}(\hat{\Theta}) = m^{-1} \sum_{i=1}^m [\partial a_{iw}(\cdot) / \partial \Theta']|_{\Theta=\hat{\Theta}} \quad \text{and} \quad \hat{V}(\hat{\Theta}) \text{ is an estimator of } V(\hat{\Theta}) = E(\hat{\Theta} - \Theta)(\hat{\Theta} - \Theta)'$$

The estimator $mse_{TR2}(\hat{\mu}_{iw})$ is approximately unbiased in the sense of Theorem 3.

Theorem 3:

$$E[mse_{TR2}(\hat{\mu}_{iw})] = MSE(\hat{\mu}_{iw}) + o(m^{-1}). \quad (17)$$

4. EMPIRICAL RESULTS

In this section, we study the performance of You – Rao MSE estimators mse_{YR1} and mse_{YR2} and the proposed MSE estimators mse_{TR1} and mse_{TR2} for the pseudo-EBLUP $\hat{\mu}_{iw}$, by applying them to a real data set given by Battese, Harter and Fuller (1988). We consider the estimation of mean hectares of corn per segment for 12 counties (areas) in north-central Iowa. The total sample size, $n = \sum_i n_i$, for the 12 counties is 36, and the sample size n_i within each county ranged from 1 to 5. The population size N_i within each county ranged from 402 to 965. The nested error model is

$$y_{ij} = \beta_0 + \beta_1 x_{1ij} + \beta_2 x_{2ij} + v_i + e_{ij}; j = 1, \dots, n_i; i = 1, \dots, m \quad (18)$$

where y_{ij} is the number of hectares of corn in the j-th segment of the i-th county, x_{1ij} and x_{2ij} are the number of pixels classified as corn and soybeans respectively, in the j-th segment of the i-th county. The small area mean $\mu_i = \beta_0 + \beta_1 \bar{X}_{1i} + \beta_2 \bar{X}_{2i} + v_i$, where \bar{X}_{1i} and \bar{X}_{2i} are the known population mean number of pixels classified as corn and soybeans per segment respectively in the i-th county, by using LANDSAT satellite readings. We constructed an artificial sampling design to compare the performance of the MSE estimators. We assumed different weights \tilde{w}_{ij} for different counties (areas) as shown in Table 1. We compute the standard errors $s = \sqrt{mse}$ reported in Table 1, using mse_{YR1} , mse_{YR2} , mse_{TR1} and mse_{TR2} given by (7), (8), (15) and (16).

Table1. Standard errors of estimators for corn

County	Sample Size	Weight	$s_{YR1}(\hat{\mu}_{iw})$	$s_{TR1}(\hat{\mu}_{iw})$	$s_{YR2}(\hat{\mu}_{iw})$	$s_{TR2}(\hat{\mu}_{iw})$
Cerro Gordo	1	1	9.96	9.66	9.33	8.83
Hamilton	1	1	9.72	9.45	9.16	8.62
Worth	1	1	9.55	9.30	10.01	8.27
Humboldt	2	(.45,.55)	8.36	8.17	8.29	7.29
Franklin	3	(.4,.25,.35)	6.71	6.54	6.97	5.98
Pocahontas	3	(.2,.5,.3)	6.94	6.85	7.26	6.29
Winnebago	3	(.4,.3,.3)	6.70	6.54	6.66	6.04
Wright	3	(.8,.1,.1)	9.23	8.93	9.01	8.02
Webster	4	(.4,.1,.45,.05)	6.93	6.80	6.85	6.26
Hancock	5	(.3,.2,.4,.03,.07)	6.26	6.14	6.03	5.63
Kossuth	5	(.2,.08,.25,.13,.34)	5.75	5.65	5.88	5.26
Hardin	5	(.2,.34,.25,.08,.13)	5.96	5.86	5.88	5.32

Table 1 suggests that the effect of estimating the cross-product term on MSE estimator is fairly small when comparing s_{YR1} to s_{TR1} . However, the area-specific version s_{TR2} seems to be significantly smaller than the corresponding s_{YR2} for some counties, for example for the county Worth: $s_{YR2} = 10.01$ and $s_{TR2} = 8.27$. Also, s_{TR2} is consistently smaller than s_{TR1} across areas.

5. SUMMARY

In this paper, we have obtained a second order approximation to the mean squared error (MSE) of the survey weighted pseudo-EBLUP estimator introduced by You and Rao (2002). Using this approximation, estimators of MSE that are nearly unbiased are also obtained. Our approximation accounts for the cross-product term in the MSE, which is ignored in You and Rao (2002).

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