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RE-CALIBRATION OF HIGHER-ORDER CALIBRATION WEIGHTS

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ABSTRACT

A new technique for re-calibrating the higher-order calibrated estimators of the variances of various estimators of the population total is proposed. Recent advances in programming techniques and computational speed make the approach appealing for practical use. Estimators of the variances of the sample mean and the ratio and regression estimators under different sampling schemes are shown to be special cases of the proposed technique. A new system of predictors for the population variance is shown to be a special case of the approach as well. The results of an empirical study designed to investigate the properties of the proposed methodology under simple sampling designs are also reported.

Keywords: Variance Estimation; Calibration; Ratio and Regression Type Predictors; Auxiliary Information.

RÉSUME

On propose une nouvelle technique pour recalibre les estimateurs de grand ordre calibres des variances de divers estimateurs du total d'une population. Les progrès récents dans les techniques de programmation et la vitesse de calcul font de cette approche une solution intéressante en pratique. Les estimateurs des variances de la moyenne échantillonnale et les estimateurs de taux et de régression sous différentes méthodes d'échantillonnage s'avèrent des cas spéciaux de la technique proposée. Un nouveau système des prédicteurs pour la variance dans la population s'avère également un cas spécial de cette approche. Les résultats d'une étude empirique conçue pour étudier les propriétés de la méthodologie proposée sous des plans d'échantillonnage simples sont également donnés.

Mot Clé: Calage, estimation de la variance, information auxiliaire, prédicteurs de type ratio et régression.

1. INTRODUCTION

Estimates of the variances of estimators serve an important role when drawing inference for finite population parameters. Such estimates can be more efficient when based on regression, ratio or product estimation approaches, since such techniques allow for the incorporation of auxiliary information. Consider a finite population $\Omega = \{1,..., i,...N\}$, from which a probability sample s $(s \in \Omega)$ is drawn with a given sampling design, $p(.)$. In other words, $p(s)$ is the probability that s is selected. The inclusion probabilities $\pi_i = \Pr(i \in s)$ and $\pi_{ij} = \Pr(i \& j \in s)$ are assumed to be strictly positive. In addition, let $\Theta_{ij} = (\pi_i \pi_j - \pi_{ij})$. If y_i is the value of the variable of interest for the i-th population unit, then the well-known Horvitz-Thompson (1952) estimator of the population total, Y, is

$$\hat{Y}_{HT} = \sum_{i \in s} d_i y_i \tag{1.1}$$

Deville and Särndal (1992) used calibration on the auxiliary total X to modify the basic sampling design weights, $d_i = 1/\pi_i$, that appear in (1.1) and suggested a new estimator

$$\hat{Y}_G = \sum_{i \in s} w_i y_i \tag{1.2}$$

where $w_i = d_i + \left(d_i q_i x_i \Big/ \sum_{i \in s} d_i q_i x_i^2 \right) \left(X - \sum_{i \in s} d_i x_i \right)$ are the calibrated weights obtained by minimizing chi-square

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distance function $\sum_{i\in s}(w_i - d_i)^2 (d_i q_i)^{-1}$ subject to a linear constraint $\sum_{i\in s} w_i x_i = X$, and leads to the GREG as:

$$\hat{Y}_G = \sum_{i\in s} d_i y_i + \left(\sum_{i\in s} d_i q_i x_i y_i \Big/ \sum_{i\in s} d_i q_i x_i^2\right)\left(X - \sum_{i\in s} d_i x_i\right) \quad (1.3)$$

Its form depends upon the choice of q_i The variance of the estimator (1.3) can be approximated as:

$$V_{YG}\left(\hat{Y}_G\right) = \frac{1}{2}\sum_{i\neq}\sum_{j\in\Omega} \Theta_{ij}\left(d_i e_i - d_j e_j\right)^2 \quad (1.4)$$

where $e_i = y_i - \beta x_i$ such that $E_m(e_i) = 0$, $V_m(e_i) = \sigma^2 v(x_i)$ and $E_m(e_i e_j) = \rho\,\sigma^2 \sqrt{v(x_i)v(x_j)}$ for $i \neq j$, $\sigma^2 > 0$. Here ρ is the correlation coefficient between successive error terms that are related according to $e_i = \rho\, e_{i-1} + u_i$ where the $u_i \sim i.i.d.\, N(0,1)$.Following Särndal, Swensson and Wretman (1992), the Yates-Grundy (1953) form for the estimator of the variance of (1.3) is

$$\hat{V}_0\left(\hat{Y}_G\right) = \frac{1}{2}\sum_{i\neq}\sum_{j\in s} d_{ij}\left(w_i \hat{e}_i - w_j \hat{e}_j\right)^2 \quad (1.5)$$

where $d_{ij} = \left(\pi_i \pi_j - \pi_{ij}\right)/\pi_{ij}$ and $\hat{e}_i = y_i - \hat{\beta} x_i$.

Singh, Horn, and Yu (1998) consider an estimator of the variance of the GREG as:

$$\hat{V}_1\left(\hat{Y}_G\right) = \frac{1}{2}\sum_{i\neq}\sum_{j\in s} w_{ij}\left(w_i \hat{e}_i - w_j \hat{e}_j\right)^2 \quad (1.6)$$

where

$$w_{ij} = d_{ij} + \left(\frac{d_{ij} q_{ij}\left(d_i x_i - d_j x_j\right)^2}{\frac{1}{2}\sum_{i\neq}\sum_{j\in s} d_{ij} q_{ij}\left(d_i x_i - d_j x_j\right)^4}\right) \times \left[V_{YG}\left(\hat{X}_{HT}\right) - \hat{V}_{YG}\left(\hat{X}_{HT}\right)\right]$$

are the calibrated weights obtained by minimizing $\frac{1}{2}\sum_{i\neq}\sum_{j\in s}\left(w_{ij} - d_{ij}\right)^2\left(d_{ij} q_{ij}\right)^{-1}$ subject to $\frac{1}{2}\sum_{i\neq}\sum_{j\in s} w_{ij}\left(d_i x_i - d_j x_j\right)^2 = V_{YG}\left(\hat{X}_{HT}\right)$, and leads to a new estimator of variance, as

$$\hat{V}_1\left(\hat{Y}_G\right) = \hat{V}_0\left(\hat{Y}_G\right) + \hat{B}\left[V_{YG}\left(\hat{X}_{HT}\right) - \hat{V}_{YG}\left(\hat{X}_{HT}\right)\right] \quad (1.7)$$

where,

$$\hat{B} = \frac{\sum_{i\neq}\sum_{j\in s} d_{ij} q_{ij}\left(w_i \hat{e}_i - w_j \hat{e}_j\right)^2\left(d_i x_i - d_j x_j\right)^2}{\sum_{i\neq}\sum_{j\in s} d_{ij} q_{ij}\left(d_i x_i - d_j x_j\right)^4}$$
$$= \hat{\mu}_{22}/\hat{\mu}_{04}\ (say),$$

$\hat{V}_{YG}\left(\hat{X}_{HT}\right) = \frac{1}{2}\sum_{i\neq}\sum_{j\in s} d_{ij}\left(d_i x_i - d_j x_j\right)^2$ and

$V_{YG}\left(\hat{X}_{HT}\right) = \frac{1}{2}\sum_{i\neq}\sum_{j\in\Omega} \Theta_{ij}\left(d_i x_i - d_j x_j\right)^2$, $X = \sum_{i\in\Omega} X_i$,and $\hat{X}_{HT} = \sum_{i\in s} d_i x_i$. Under simple random sampling without replacement (SRSWOR), it is easy to show that the estimators of S_y^2 due to Isaki (1983), $s_I^2 = s_y^2\left(S_x^2/s_x^2\right)$, and $s_{lr}^2 = s_y^2 + \hat{\gamma}\left(S_x^2 - s_x^2\right)$, where $\hat{\gamma} = \sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2\left(x_i - \bar{x}\right)^2 \Big/ \sum_{i=1}^{n}\left(x_i - \bar{x}\right)^4$ are special cases of (1.7). On the other hand, Shah and Patel (1996) introduced a system of predictors of S_y^2 as

$$Q_{greg} = s_y^2 + \hat{\gamma}_1\left(S_x^2 - s_x^2\right) + \hat{\gamma}_2\left(\frac{1}{N}\sum_{i\in\Omega} v(x_i) - \frac{1}{n}\sum_{i\in s} v(x_i)\right) \quad (1.8)$$

where $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are sample-dependent constants.

2. RE-CALIBRATED ESTIMATOR OF THE VARIANCE OF GREG

We consider here an estimator of the variance of GREG as

$$\hat{V}_2\left(\hat{Y}_G\right) = \frac{1}{2}\sum_{i\neq}\sum_{j\in s} w_{ij}^{\circ}\left(w_i \hat{e}_i - w_j \hat{e}_j\right)^2 \quad (2.1)$$

where w_{ij}° are re-calibrated weights such that chi-square type distance function

$$D^{\circ} = \frac{1}{2}\sum_{i\neq}\sum_{j\in s}\left(w_{ij}^{\circ} - w_{ij}\right)^2\left(c_{ij} w_{ij}\right)^{-1} \quad (2.2)$$

is minimized subject to the calibration constraint

$$E_m\left[\frac{1}{2}\sum_{i\neq}\sum_{j\in s} w_{ij}^{\circ}\left(w_i \hat{e}_i - w_j \hat{e}_j\right)^2\right] = E_m V_{YG}\left(\hat{Y}_G\right)$$

or

$$\frac{1}{2}\sum_{i\neq j\in s}\sum w_{ij}^{\circ}\left(w_i^2 v(x_i) + w_j^2 v(x_j) - 2\rho w_i w_j \sqrt{v(x_i)v_j(x_j)}\right)$$
$$= \frac{1}{2}\sum_{i\neq j\in\Omega}\sum \Theta_{ij}\left(d_i^2 v(x_i) + d_j^2 v(x_j) - 2\rho d_i d_j \sqrt{v(x_i)v(x_j)}\right) \quad (2.3)$$

where c_{ij} are real constants. Optimization (2.2) subject to (2.3) yields the re-calibrated weights

$$w^{\circ}_{ij} = w_{ij} + \frac{c_{ij} w_{ij}\left(w_i^2 v(x_i) + w_j^2 v(x_j) - 2\rho w_i w_j \sqrt{v(x_i)v(x_j)}\right)}{\frac{1}{2}\sum_{i\neq}\sum_{j\in s} c_{ij} w_{ij}\left(w_i^2 v(x_i) + w_j^2 v(x_j) - 2\rho w_i w_j \sqrt{v(x_i)v(x_j)}\right)^2} \times \left\{V_{ma} - \hat{V}_{ma1}\right\} \quad (2.4)$$

Substitution of (2.4) into (2.1) provides a new estimator of the variance of GREG as

$$\hat{V}_2\left(\hat{Y}_G\right) = \hat{V}_0\left(\hat{Y}_G\right) + \hat{B}\left[V_{YG}\left(\hat{X}_{HT}\right) - \hat{V}_{YG}\left(\hat{X}_{HT}\right)\right] + \hat{D}\left[V_{ma} - \hat{V}_{ma_1}\right] \quad (2.5)$$

where

$$\hat{B} = \frac{\sum_{i\neq}\sum_{j\in s} d_{ij} q_{ij}\left(d_i x_i - d_j x_j\right)^2 \left(w_i \hat{e}_i - w_j \hat{e}_j\right)^2}{\sum_{i\neq}\sum_{j\in s} d_{ij} q_{ij}\left(d_i x_i - d_j x_j\right)^4},$$

$$\hat{D} = \left(\sum_{i\neq}\sum_{j\in s} c_{ij} d_{ij}\left(w_i^2 v(x_i) + w_j^2 v(x_j) - 2\rho w_i w_j \sqrt{v(x_i)v(x_j)}\right)\left(w_i e_i - w_j e_j\right)^2\right) \times \left(\sum_{i\neq}\sum_{j\in s} c_{ij} w_{ij}\left(w_i^2 v(x_i) + w_j^2 v(x_j) - 2\rho w_i w_j \sqrt{v(x_i)v(x_j)}\right)^2\right)^{-1}$$

$$\hat{V}_{YG}\left(\hat{X}_{HT}\right) = \frac{1}{2}\sum_{i\neq}\sum_{j\in s} d_{ij}\left(d_i x_i - d_j x_j\right)^2,$$

$$V_{ma} = \frac{1}{2}\sum_{i\neq}\sum_{j\in} \Theta_{ij}\left(d_i^2 v(x_i) + d_j^2 v(x_j) - 2\rho\, d_i d_j \sqrt{v(x_i)v(x_j)}\right),$$

and

$$\hat{V}_{ma1} = \frac{1}{2}\sum_{i\neq}\sum_{j\in s} w_{ij}\left(w_i^2 v(x_i) + w_j^2 v(x_j) - 2\rho\, w_i w_j \sqrt{v(x_i)v(x_j)}\right).$$

The class (2.5) is wider than Wu (1982), Deville and Sarndal (1992), Deng and Wu (1987), Shah and Patel (1996), Singh, Horn and Yu (1998) and Wu and Sitter (2001) and we named it higher order model assisted calibration.

3. RE-CALIBRATION USING OPTIMAL DESIGNS FOR THE GREG-ESTIMATOR

Consider the situation of ρ not necessarily zero and note that if $\pi_i \propto \sqrt{v(x_i)}$, then (2.3) reduces to

$$\sum_{i\neq}\sum_{j\in s} w^{\circ}_{ij}\left(h_i^2 + h_j^2 - 2\rho\, h_i h_j\right) = 2(1-\rho)\sum_{i\neq j}\sum_{\in\Omega} \Theta_{ij} \quad (3.1)$$

where $h_i = 1 + \left(q_i x_i \Big/ \sum_{i\in s} d_i q_i x_i^2\right)\left[X - \hat{X}_{HT}\right]$. The re-calibrated estimator of variance takes the form

$$\hat{V}_2\left(\hat{Y}_G\right) = \frac{1}{2}\sum_{i\neq}\sum_{j\in s} w_{ij}\left(w_i \hat{e}_i - w_j \hat{e}_j\right)^2 + \frac{\sum_{i\neq}\sum_{j\in s} w_{ij} c_{ij}\left(h_i^2 + h_j^2 - 2\rho h_i h_j\right)\left(w_i e_i - w_j e_j\right)^2}{\sum_{i\neq}\sum_{j\in s} c_{ij} w_{ij}\left(h_i^2 + h_j^2 - 2\rho h_i h_j\right)^2} \times \left[(1-\rho)\sum_{i\neq j}\sum_{\in s} \Theta_{ij} - \frac{1}{2}\sum_{i\neq}\sum_{j\in s} w_{ij}\left(h_i^2 + h_j^2 - 2\rho h_i h_j\right)^2\right] \quad (3.2)$$

This result illustrates that the proposed technique works to re-calibrate the Yates and Grundy (1953) form of the estimator of the variance of GREG under the condition of minimum variance for the estimator of the total under the true model. It is often the case that $c_{ij} = 1$. If this is so, then (3.2) is a new estimator of variance of the GREG estimator. Three other special cases of (3.2) are also worthy of note.

Case 3.1. If $\rho = 0$ and $c_{ij} = \left(h_i^2 + h_j^2\right)^{-1}$, then the estimator (3.2) reduces to

$$\hat{V}_2\left(\hat{Y}_G\right) = \frac{1}{2}\sum_{i\neq}\sum_{j\in s} w_{ij}\left(w_i \hat{e}_i - w_j \hat{e}_j\right)^2 \left\{\sum_{i\neq j}\sum_{\in\Omega} \Theta_{ij} \Big/ \sum_{i\neq j}\sum_{\in s} w_{ij}\left(h_i^2 + h_j^2\right)\right\} \quad (3.3)$$

Case 3.2. If $\rho \in [-1, 0) \cup (0, 1)$ and $c_{ij} = \left(h_i^2 + h_j^2 - 2\rho\, h_i h_j\right)^{-1}$ then (3.2) takes the form

$$\hat{V}_2\left(\hat{Y}_G\right) = \frac{1}{2}\sum_{i\neq}\sum_{j\in s} w_{ij}\left(w_i \hat{e}_i - w_j \hat{e}_j\right)^2 \times \left\{2(1-\rho)\sum_{i\neq j}\sum_{\in\Omega} \Theta_{ij} \Big/ \sum_{i\neq j}\sum_{\in s} w_{ij}\left(h_i^2 + h_j^2 - 2\rho\, h_i h_j\right)\right\} \quad (3.4)$$

Case 3.3. If $\rho = +1$, then for $c_{ij} = \left(h_i - h_j\right)^{-2}$ the estimator of variance in (3.2) becomes

$$\hat{V}_2\left(\hat{Y}_G\right) = 0. \quad (3.5)$$

Note also that the condition $\pi_i \propto \sqrt{v(x_i)}$ corresponds to the Godambe and Joshi (1965) lower bound of variance, so the variance for fixed sample design under the true model may be equal to zero. This demonstrates the usefulness of the proposed re-calibration method of the estimator of the variance of the GREG estimator.

4. EMPIRICAL STUDY: NO AUTOCORRELATION

Under the assumption that $\rho = 0$, the performance of the re-calibrated estimator in each case has been assessed at a number of values for the model parameter *g* with $v(x_i) = x_i^g$. To avoid any confusion, we have redefined the estimators considered for comparison in the empirical study. Note that Singh, Horn, and Yu (1998) conducted a similar comparison of low and high level calibration approaches.

4.1. Ratio estimator

We compare the estimator of the variance of the ratio estimator,

$$\hat{V}_1\left(\hat{Y}_{Ratio}\right) = \frac{N^2(1-f)}{n(n-1)} \sum_{i=1}^{n} \hat{e}_i^2 \left(\frac{X}{\hat{X}}\right)^2 \left(S_x^2 / s_x^2\right),$$

with the re-calibrated estimator of variance

$$\hat{V}_2\left(\hat{Y}_{Ratio}\right) = \hat{V}_1\left(\hat{Y}_{Ratio}\right) + \frac{N^2(1-f)}{n}\hat{\gamma}$$
$$\times\left\{\frac{1}{N}\sum_{i=1}^{N} x_i^g - \frac{1}{n}\sum_{i=1}^{n} x_i^g \left(\frac{X}{\hat{X}}\right)^2 \left(S_x^2 / s_x^2\right)\right\}$$

where $\hat{\gamma} = \sum_{i\neq}^{n}\sum_{j=1}^{n}\left(x_i^g + x_j^g\right)\left(\hat{e}_i - \hat{e}_j\right)^2 \Big/ \left(\sum_{i\neq}^{n}\sum_{j=1}^{n}\left(x_i^g + x_j^g\right)\right)^2$.

4.2. Regression Estimator

Finally, we compare the estimator of the variance of the regression estimator

$$\hat{V}_1\left(\hat{Y}_{greg}\right) = \frac{N^2(1-f)}{n(n-1)} \sum_{i=1}^{n} \hat{e}_i^2 + \hat{\gamma}\left(X - \hat{X}\right)$$
$$+ \hat{\gamma}_2\left(X - \hat{X}\right)^2 + \hat{\gamma}_3\left(S_x^2 - s_x^2\right)$$

with the re-calibrated estimator of the variance of GREG

$$\hat{V}_2\left(\hat{Y}_{greg}\right) = \hat{V}_1\left(\hat{Y}_{greg}\right)\left(\frac{1}{N-1}\frac{\sum_{i=1}^{N} x_i^g}{\frac{1}{n-1}\sum_{i=1}^{n} t_i^2 x_i^g + \hat{c}_0\left(S_x^2 - s_x^2\right)}\right),$$

where

$$\hat{c}_0 = \sum_{i\neq}^{n}\sum_{j=1}^{n}\left(x_i - x_j\right)^2\left(t_i^2 x_i^g + t_j^2 x_j^g\right) \Big/ \sum_{i\neq}^{n}\sum_{j=1}^{n}\left(x_i - x_j\right)^4 \quad \text{and}$$

$t_i = 1 + n x_i\left(\bar{X} - \bar{x}\right) \Big/ \sum_{i=1}^{n} x_i^2$.

We make use of a population consisting of $N = 20$ units from Horvitz and Thompson (1952), the study variable, y, is the number of households on the *i-th* block, while the known auxiliary character, x, is the eye estimated number of households on the *i-th* block. All possible samples of size $n = 5$ were selected by SRSWOR, which results in $\binom{N}{n} = 15504 = M$ samples. For the *k-th* sample, the ratio estimator $\hat{Y}_{Ratio}\,|_k$ and its variance, $V\left(\hat{Y}_{Ratio}\right) = M^{-1}\sum_{k=1}^{M}\left[\hat{Y}_{Ratio}\,|_k - Y\right]^2$, were computed, along with the higher-order and re-calibrated estimators of variance $\hat{V}_h\left(\hat{Y}_{Ratio}\right)|_k$, $h = 01,2$ respectively, values of *g* ranging from –1 to 3 in steps of 0.2 were used. The mean squared error was computed as $MSE\left\{\hat{V}\left(\hat{Y}_{Ratio}\right)\right\} = M^{-1}\sum_{k=1}^{M}\left[\hat{V}_h\left(\hat{Y}_{Ratio}\right)|_k - V\left(\hat{Y}_{Ratio}\right)\right]^2$ were then evaluated over all possible samples. The percent relative efficiency of $\hat{V}_2\left(\hat{Y}_{Ratio}\right)$ with respect to $\hat{V}_1\left(\hat{Y}_{Ratio}\right)$ was calculated as $RE = MSE\left\{\hat{V}_1\left(\hat{Y}_{Ratio}\right)\right\}\times 100 / MSE\left\{\hat{V}_2\left(\hat{Y}_{Ratio}\right)\right\}$ while the coverage by 95% confidence intervals $CCI\left[\hat{V}_h\left(\hat{Y}_{Ratio}\right)\right]$ was determined for the *h-th* estimator by determining the proportion of times that the true population total, Y, fell between the limits defined by $\hat{Y}_{Ratio}\,|_k \mp t_{n-h-2}(\alpha)\sqrt{\hat{V}_h\left(\hat{Y}_{Ratio}\right)|_k}$. The relative biases in the estimators were also computed and are presented in Table 1. This entire process was then repeated for the regression estimators

$$\hat{Y}_{greg}\,|_k = \hat{Y} + \left(\sum_{i=1}^{n} x_i y_i \Big/ \sum_{i=1}^{n} x_i^2\right)\left(X - \hat{X}\right).$$

The simulation study shows that the proposed estimator is always better than the existing estimators.

5. EMPIRICAL STUDY: AUTOCORRELATED ERROR TERMS

In order to consider the situation where ρ is not necessarily zero, for different pairs of *g* and ρ as shown in Table 2 we generated using IMSL subroutines in FORTRAN finite populations of *N* = 1000 units each from the model $y_i = \beta x_i + \sqrt{x_i^g}\, e_i$ where $e_i = \rho e_{i-1} + u_i$ with $e_0 = 0$ and $u_i \sim N(0,1)$. The auxiliary variable x_i was assumed to follow a beta distribution with both parameters set at 0.1. The *Y* values were obtained from the model with $\beta = 1.2$. From each population of 1000 units, we selected 5000 samples of *n* = 50. Then, for each population defined by a different pair of g and ρ, we determined the efficiency of the proposed estimator of the variance of the ratio estimator

$$\hat{V}_2\left(\hat{Y}_{Ratio}\right)=\frac{N^2(1-f)}{n}\left[\frac{1}{(n-1)}\sum_{i=1}^{n}\hat{e}_i^2\left(\frac{X}{\hat{X}}\right)^2\left(S_x^2/s_x^2\right)\right.$$

$$\left.+\hat{\gamma}\left\{\frac{1}{N}\sum_{i=1}^{N}x_i^g-cf-\left\{\frac{1}{n}\sum_{i=1}^{n}x_i^g-cf\right\}\left(\frac{X}{\hat{X}}\right)^2\left(S_x^2/s_x^2\right)\right\}\right]$$

where

$$\hat{\gamma}=\frac{\sum_{i\neq}^{n}\sum_{j=1}^{n}\left(x_i^g+x_j^g-2\rho\sqrt{x_i^g x_j^g}\right)\left(e_i-e_j\right)^2}{\sum_{i\neq}^{n}\sum_{j=1}^{n}\left(x_i^g+x_j^g-2\rho\sqrt{x_i^g x_j^g}\right)^2}$$

and $cf=\rho\left\{\sum_{i=1}^{N}\sqrt{x_i^g}\right\}^2\Big/\left(N(N-1)\right)$ relative to $\hat{V}_1\left(\hat{Y}_{Ratio}\right)$.

The results, presented in Table 2, show that the relative efficiencies range from 109.3% to 410.8% with a median of 119.5%. Finally, note that the proposed estimator is not defined for $\rho=1$ and $g=0$.

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Table 1. Comparison of the estimators of the variance of Ratio and GREG based on based on the Horvitz and Thompson (1952) population for n=5 and N=20.

		Ratio						GREG				
g		$RB\{\hat{V}_1\}$	$RB\{\hat{V}_2\}$	RE	$C\{\hat{V}_1\}$	$C\{\hat{V}_2\}$		$RB\{\hat{V}_1\}$	$RB\{\hat{V}_1\}$	RE	$C\{\hat{V}_1\}$	$C\{\hat{V}_2\}$
-1.0		0.204	0.069	170.5	0.915	0.843		0.009	0.054	53.65	0.925	0.954
-0.8			0.052	179.7		0.839			0.031	59.01		0.954
-0.6			0.035	188.8		0.837			0.010	64.38		0.953
-0.4			0.020	197.7		0.834			-0.008	69.76		0.952
-0.2			0.005	206.6		0.832			-0.024	75.12		0.952
0.0			-0.009	215.7		0.834			-0.039	80.47		0.953
0.2			-0.023	225.1		0.835			-0.052	85.81		0.954
0.4			-0.036	234.8		0.836			-0.063	91.14		0.954
0.6			-0.049	244.8		0.840			-0.073	96.45		0.954
0.8			-0.061	255.1		0.846			-0.081	101.7		0.954
1.0			-0.072	265.6		0.852			-0.088	106.9		0.955
1.2			-0.082	275.9		0.859			-0.093	112.1		0.956
1.4			-0.090	286.0		0.867			-0.097	117.2		0.957
1.6			-0.096	295.7		0.877			-0.100	122.2		0.957
1.8			-0.100	304.6		0.887			-0.101	126.9		0.958
2.0			-0.101	312.7		0.897			-0.100	131.4		0.958
2.2			-0.100	319.8		0.906			-0.098	135.5		0.958
2.4			-0.096	325.7		0.915			-0.094	139.2		0.958
2.6			-0.089	330.3		0.922			-0.088	142.4		0.958
2.8			-0.079	333.4		0.928			-0.080	144.9		0.958
3.0			-0.066	334.9		0.934			-0.069	146.7		0.959

Table 2. Relative efficiency of the proposed estimator of variance of the ratio estimator when autocorrelation (ρ) is not necessarily zero for *N=1000* and *n=50*.

	ρ				
g	0.0	0.25	0.50	0.75	1.00
0.0	410.8	397.9	285.2	311.3	-
0.5	148.3	152.4	162.0	158.7	109.3
1.0	126.9	130.7	136.8	136.5	109.4
1.5	120.2	123.7	128.6	129.1	112.3
2.0	116.7	119.9	124.1	124.9	115.4
2.5	114.5	117.4	121.2	122.1	117.5
3.0	112.9	115.6	119.1	120.2	118.5
3.5	111.7	114.3	117.5	118.7	118.7
4.0	110.7	113.2	116.3	117.5	118.4